B.A./B.Sc. SECOND YEAR MATHEMATICS

SEMESTER-III,PAPER-III

MODEL QUESTION PAPER

ABSTRACT ALGEBRA

Time: 3 Hours *Max. Marks : 75*

PART - A

I. Answer any FIVE of the following Questions : 5 X 5= 25 Marks

1. Prove that in a group G Inverse of any Element is unique.

2. $G=\{1,2,3,4,5,6\}$ Prepare composition table and prove that G is a finite abelian group of order 6 with respect to X_7.

3. If H is any subgroups of G then prove that $H^{-1}=H$.

4. State and prove Lagrange's theorem.

5. Prove that intersection of any two normal subgroup is again a normal subgroup.

6. Prove that the homomorphic image of a group is a group.

7. If $A=\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}, B=\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$ find AB and BA.

8. Find the inverse of the permutation: $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 3 & 4 & 5 & 6 & 1 & 2\end{pmatrix}$

9. Define Types of rings and give one example for each.

10. If R is a Boolean ring then prove that $a+a=0 \forall a \in R$.

PART - B

Answer any FIVE of the following Questions. 5 $\times$ 10 =50 Marks

SECTION - A

11. Define abelian group. Prove that the set of n^{th} roots of unity under multiplication form a finite abelian group.

12. Show that the set of all positive rational numbers form on abelian group under the composition '0' defined by $aob=\frac{ab}{2}$.

13. Prove that a non-empty finite subset of a group which is closed under multiplication is a subgroup of G.

14. Prove that the union of two subgroups of a group is a subgroup if f one is contained in the other.

15. A subgroup H of G is normal if and only if $xHx^{-1}=H$.

16. State and prove fundamental theorem on Homomorphism of Groups.

17. Examine the following permutation are even (or) odd

(i) $f=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$ (ii) $g=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 \end{pmatrix}$

18. Find the regular permutation group isomorphic to the multiplicative group $G=\{1,-1,i,-i\}$ where $i^2=-1$.

19. Prove that a finite integral domain is a field.

20. State and Prove Cancellation laws on Rings.