B.A./B.Sc. THIRD YEAR MATHEMATICS SYLLABUS

SEMESTER-IV,PAPER-V

MODEL QUESTION PAPER

LINEAR ALGEBRA

Time: 3 Hours ***Max. Marks : 75***

PART - A

I. Answer any FIVE of the following Questions : ***5 X 5= 25 Marks***

1. Prove that intersection of two subspaces is again a subspace.

2. Show that the system of vector $(1,3,2),(1,-7,-8),(2,1,-1)$ of $V_3(R)$ is Linearly dependent.

3. State and prove "Invariance theorem".

4. Show that the vectors $(1,1,2),(1,2,5),(5,3,4)$ of $R^3(R)$ do not form a basis set of $R^3(R)$.

5. Show that the mapping $T:V_3(R)\to V_2(R)$ is defined by $T:(x,y,z)=(x-y,x-z)$ is a Linear Transformation.

6. $T:V_3(R)\to V_2(R)$ and $H:V_3(R)\to V_2(R)$ be two Linear Transformations $T(x,y,z)=(x-y,y+z)$ and $H(x,y,z)=(2x,y-3)$ Find (i) H+T (ii) aH.

7. Obtain the rank of the matrix $A=\begin{bmatrix}-1 & 2 & 0\\ 3 & 7 & 1\\ 5 & 9 & 3\end{bmatrix}$.

8. Show that the equations $x+y+z-3=0,\ 3x-5y+2z-8=0,\ 5x-3y+4z-14=0$ are consistent.

9. State and prove Triangle Inequality.

10. If α,β are two vectors in Euclidean space $V(R)$ such that $\|\alpha\|=\|\beta\|$ prove that $(\alpha+\beta,\alpha-\beta)=0$.

PART - B

Answer any <u>FIVE</u> of the following Questions. ***$5 \times 10 = 50$ Marks***

11. If $V(F)$ be a vector space. $\omega \subseteq V$. Prove that the necessary and sufficient conditions for ω to be a subspace of V are

(i) $\alpha \in \omega, \beta \in \omega \Rightarrow \alpha - \beta \in \omega$

(ii) $a \in F, \alpha \in \omega \Rightarrow a\alpha \in \omega$.

12. If show that are the sub sets of a vector space $v(F)$ then prove that $L(S \cup T) = L(S) + L(T)$.

13. State and prove Basis Existence theorem.

14. Find the co-coordinators of (2,3,4,-1) with respect to the basis of $V_4(R)$ B={(1,1,1,2),(1,-1,0,0),(0,0,1,1),(0,1,0,0)}

15. Find $T(x, y, z)$ where $T: R^3 \to R$ is defined by $T(1,1,1) = 3$, $T(0,1,-2) = 1$, $T(0,0,1) = -2$.

16. Define Null space. Prove that Null space $N(T)$ is subspace of $U(F)$ where $T: U \to V$ is a Linear Transformation.

17. If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ verify cayley – Hamilton theorem. Hence find A^{-1}.

18. Find the characteristic roots and vectors to the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

19. State and prove parallelogram Law.

20. If α, β and two vectors in an I.P.S. then prove that α, β are Linear Independent iff $|(\alpha, \beta)| = \|\alpha\| \|\beta\|$.