B.A./B.Sc. SECOND YEAR MATHEMATICS

SEMESTER-IV,PAPER-IV

MODEL QUESTION PAPER

REAL ANALYSIS

Time: 3 Hours *Max. Marks : 75*

PART - A

I. Answer any FIVE of the following Questions : 5 X 5= 25 Marks

1. Test for convergence $\sum \frac{1}{n^2+1}$.

2. State Cauchy's root test and test for convergence $\sum \left(1+\frac{1}{n}\right)^{-n^2}$.

3. Discuss various types of discontinuity.

4. Examine for continuity of a function $f(n)=|x|+(x-1)$ at x=0.

5. If $f(x)=\frac{x}{1+e^{\frac{1}{x}}}$ if $x \neq 0$ and $f(x)=0$ if x=0 show that f is not derivable at x = 0.

6. Prove that $f(x)=x^2 \sin\left(\frac{1}{x}\right), x \neq 0$ and $f(0)=0$ is derivable at the origin.

7. State cauchy's Mean value theorem.

8. Find 'C' of the Lagrange's mean value theorem for $f(x)=(x-1)(x-2)(x-3)$ on $[0,4]$.

9. If $f(x)=x^2$ on $[0,1]$ and $P=\left\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\right\}$ compute $L(P,f)$ and $U(P,f)$.

10. Prove that a constant function is Riemann integrable on $[a,b]$.

PART - B

Answer any FIVE of the following Questions. ***5 × 10 =50 Marks***

11. State and Prove Root Test.

12. State and prove Ratio Test.

13. Discuss the continuity of $f(x)=\dfrac{x\left(e^{\frac{1}{x}}-e^{-\frac{1}{x}}\right)}{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}$ for $x\neq 0$ and $f(0)=0$ at x = 0.

14. If f is continuous on $[a,b]$ and $f(a), f(b)$ having opposite sign then prove that there exit $C\in(a,b)\ni f(c)=0$.

15. Show that $f(x)=x\sin\left(\frac{1}{x}\right), x\neq 0, f(x)=0$ when x=0 is continuous but not derivable at x=0.

16. Show that $f(x)=\dfrac{x\left(e^{\frac{1}{x}}-1\right)}{e^{\frac{1}{x}}+1}$ if $x\neq 0$ and $f(0)=0$ is continuous at x=0 but not derivable at x=0.

17. State and prove Rolle's theorem.

18. Using Lagrange's theorem show that $x>\log(1+n)>\frac{x}{1+x}$ if $f(x)=\log(1+x)$.

19. If $f:[a,b]\to R$ is monotonic on $[a,b]$ then f is integrable on $[a,b]$.

20. If $f\in R[a,b]$ and m, M are the infimum and supremum of f on $[a,b]$, then

$$m(b-a)\leq\int_a^b f(x)\,dx\leq M(b-a).$$